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INTRODUCTION

Considerable interest has developed in the last few years in developing an optimum sequence for removing ore and waste from an open-pit mine. This interest has developed for two reasons: 1) the sequence in which ore is removed from a deposit influences the length of the pay-back period and the internal rate of return, and 2) the increased expertise in the mining industry in computer applications.

The adage "high-grade first" is not as easily implemented in an open-pit mine as in an underground mine because of the time required to expose an ore zone. For this reason, planning an open-pit mining sequence is much more difficult than developing an underground mining plan. The objective in developing an open-pit mining sequence – to maximize the present worth of the deposit– is generally agreeable to all people involved. The constraints that must be followed by the mining sequence are a little more difficult to agree upon and will vary from mine to mine. The constraints imposed by pit wall stability are obvious: the mining sequence cannot produce a pit wall having a slope greater than that which is stable. Next, are constraints on the minimum tonnage of the various classifications of ore that must be produced during any one time period to satisfy the demand from the mill and leach dumps. Finally, there are constraints to insure that a roadway is maintained; and similarly, constraints that insure orderly mining of the deposit. Davis and Williams (1973) list 13 constraints to the mining sequence.

The optimum mining sequence problem is such that integer programming is the first technique which comes to mind when this problem is investigated (for a discussion on integer programming see Principles of Operations Research by H. M. Wagner, Prentice Hall, Inc., 1969, p. 445-511). If, in the mining sequence matrix (M(I, J), where I represents the number of the mining period and J represents the number of the block of ore), zero is assigned to the value of the blocks not mined in a specified mining period and 1 is assigned to the value of the block that is mined in that period, the matrix will describe the mining sequence. A 1 in the matrix element M(5, 20) would indicate that block 20 is mined in the 5th mining period. All other elements M(i, 20) would be zero by necessity inasmuch as block 20 can only be mined once. Likewise all elements M(5, j) would be zero if only one block could be mined in any one period. This formulation of the mining sequence problem is correct in theory. However, integer programming problems of this complexity do not readily lend themselves to optimum solutions. Although the problem can be solved for a "good" mining sequence, the problem cannot be solved for the optimum mining sequence (that sequence which produces the maximum present value of the deposit).

DETERMINATION OF THE MINING SEQUENCE

The approach used in this investigation of the mining sequence problem is termed "set dynamic programming". To facilitate a description of this procedure a simplified example will be used. The example is simplified in that an ore deposit of only 2 dimensions (height and width) is considered, and, in that the only constraints imposed on the mining sequence are 1) that the pit wall slope not exceed 1:1 at any point, and 2) that each mining level be entered only at one point.

Fig. 1 is a schematic of the 2-dimensional ore deposit. In each block is written the block index number and the net value of that block. The net value would be determined by subtracting from the mill receipts for that block the mining and processing cost. The mining costs are the costs for that block only and do not include stripping costs. At this point, a decision does not have to be made as to what blocks will, or will not, be mined, but a decision is needed in determining which blocks will be sent to the mill, to the leach dump, or to the waste dump. A decision also must be made as to which will be the last block in the sequence. These decisions do not present problems, however, if the last block is selected at the bottom vertice of the inverted triangle containing all blocks of positive value. An alternative procedure would be to select a hypothetical block (a block that does not exist) as the last block.

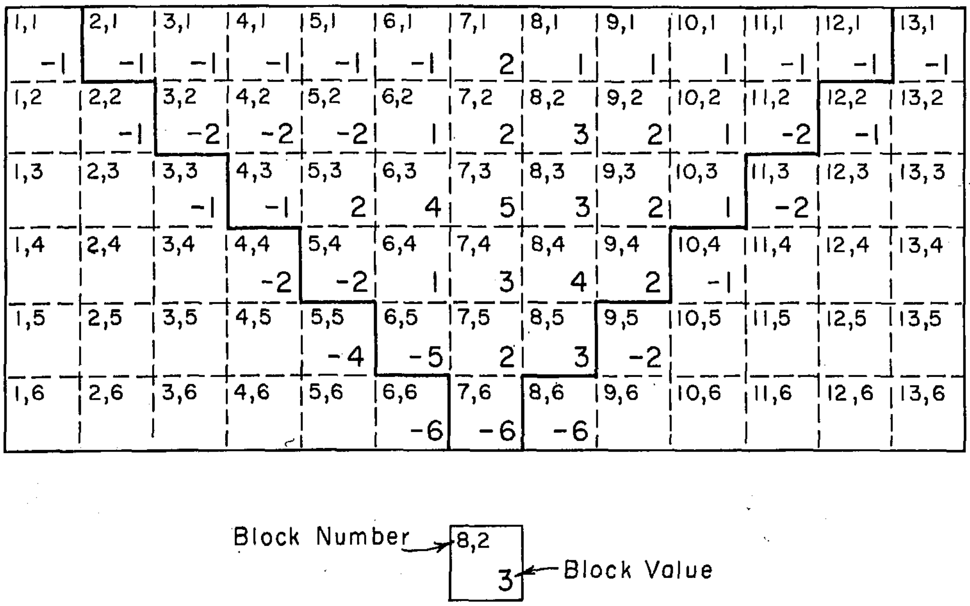

FIGURE 1 – Schematic of the ore deposit showing block numbers and block values.

In this example block 7, 6 is selected as the last block to be mined. (Obviously block 7, 6, however, will never be mined because it has a negative net value.) Including block 7, 6 at the bottom of the triangle of the blocks containing the ore deposit, there are 36 blocks which can be mined in accordance with the first constraint. If the mining life is divided into 36 time periods (of unspecified length), then 36 time periods are needed to mine all the blocks. We can now start to develop the possible mining sequences, starting in the 36th period with the last block to be mined: block 7, 6. In the 35th period are 2 blocks that possibly can be mined: 6, 5 and 8, 5. Mining block 7, 5 would violate the second constraint and mining any other block will produce a situation which violates the first constraint. The 2 possible mining sequences for the last 2 periods (35th and 36th) are then

6, 5 – 7, 6
and 8, 5 – 7, 6

In the 34th period are several choices for the block to be mined, depending on the block mined in the 35th period. If the last two blocks mined are 6, 5 and 7, 6 then the possible sequences for the last 3 periods are

1) 7, 5 – 6, 5 – 7, 6
2) 8, 5 – 6, 5 – 7, 6
3) 5, 4 – 6, 5 – 7, 6

And if the last 2 blocks mined are 8, 5 and 7, 6 then the possible sequences for the last 3 periods are

4) 6, 5 – 8, 5 – 7, 6
5) 7, 5 – 8, 5 – 7, 6
6) 9, 4 – 8, 5 – 7, 6

Six sequences for mining the last 3 blocks in the deposit are now possible. However, sequences 2 and 4 are different permutations of the same combination. That is, they represent mining the same 3 blocks but in a different order. Inasmuch as they are alternatives for doing the same job, an economic evaluation can be performed on each alternative, and the best alternative selected. The alternative which is less attractive can be dropped from further consideration. This procedure is as follows. A reference time zero (for the purpose of discounting the cash flows) is first selected as the beginning of the present period (period 34). Then with end of period payments, the resulting cash flows for the 34th, 35th and 36th blocks mined are discounted 1, 2 and 3 periods, respectively. Referring to fig. 1 for the net values assigned to the blocks, and selecting a discounting rate of 10 percent per period, the present values of the sequence are:

Sequence	Present Value
2	-5.91
4	-6.57

As a result of this discounted cash flow analysis, if the last three blocks to be mined were 7, 6, 6, 5 and 8, 5 then sequence 2 would be chosen over alternative sequence 4 because the resulting cash flow is more positive. By eliminating sequence 4 from the six possible sequences for the last three blocks to be mined, only five sequences need be considered in the remaining determinations of the optimum sequence for mining the 36 blocks:

7, 5 – 6, 5 – 7, 6
8, 5 – 6, 5 – 7, 6
5, 4 – 6, 5 – 7, 6
7, 5 – 8, 5 – 7, 6
9, 4 – 8, 5 – 7, 6

The procedure for determining the sequences for the last four blocks to be mined is the same as for the last three blocks. Eliminating duplicate combinations by the present value analysis gives the following choices for the last four blocks to be mined.

Sequence No.	
1	8, 5 – 7, 5 – 6, 5 – 7, 6
2	9, 4 – 8, 5 – 6, 5 – 7, 6
3	7, 5 – 5, 4 – 6, 5 – 7, 6
4	8, 5 – 5, 4 – 6, 5 – 7, 6
5	4, 3 – 5, 4 – 6, 5 – 7, 6
6	9, 4 – 7, 5 – 8, 5 – 7, 6
7	10, 4 – 9, 4 – 8, 5 – 7, 6

The procedure outlined is repeated for each time period until a sequence containing all 36 blocks is developed. Because the 36th period sequences contain all 36 blocks, all of the sequences will be permutations of the same combination; consequently, the one optimum sequence can be determined. Fig. 2 is the schematic of the deposit with the blocks numbered in the order they are to be removed to maximize the present value of the deposit.

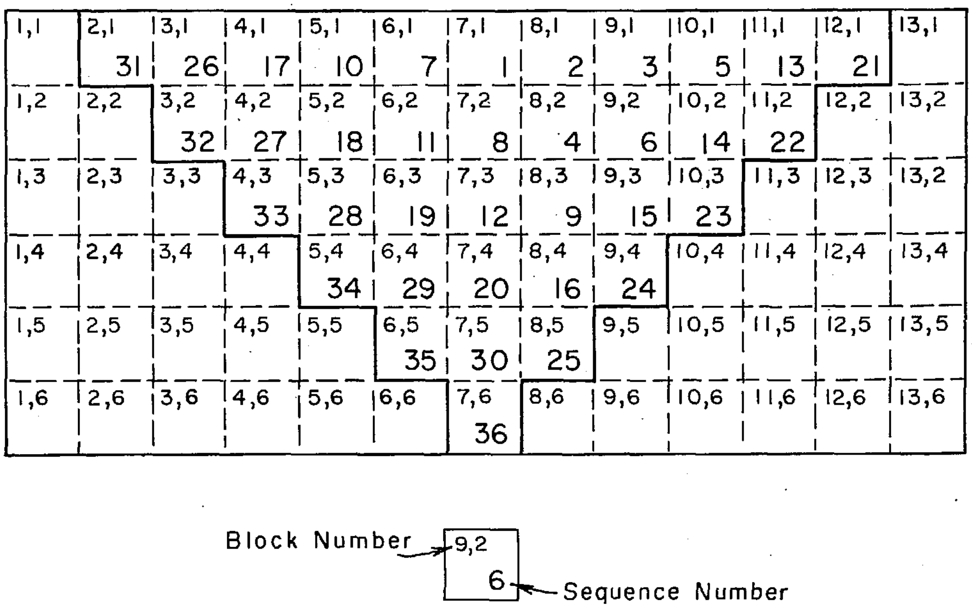

FIGURE 2 – Schematic of the deposit showing the optimum sequence for removing blocks.

DETERMINATION OF THE PIT OUTLINE

Once the optimum sequence is developed, the pit outline can be developed. Obviously, the last six blocks in the sequence do not produce a profit and would not be mined. Block 7, 5 is the last block. The procedure for determining which of the 36 blocks in the optimum sequence are actually mined is as follows:

1) Determine which is the last block in the optimum sequence with a positive value. Drop all blocks after this one (they will all have negative values).
2) Determine the present worth (any arbitrary time zero) for the last remaining negative block (or set of negative blocks) and all the blocks between that block and the last block in the sequence.
3) If the present value of this subsequence is negative, drop all the blocks from the optimum sequences and repeat step 2. If the subsequence has a positive present value, replace the subsequence by an equivalent block value at the end of the sequence.
4) Repeat step 2 and 3 until the first mined block is included in the subsequence.

In the example deposit, blocks 7, 6 – 6, 5 – 5, 4 – 4, 3 – 3, 2 and 2, 1 are dropped by inspection. The next subsequence consists of blocks 3, 1 – 4, 2 – 5, 3 – 6, 4 and 7, 5. The net present value (beginning of period 26 taken as time zero) of this subsequence discounted 10 percent per period is +0.89; consequently, they are kept in the optimum sequence. These five blocks now are combined (figuratively) into the last block in the next subsequence: 12, 1 – 11, 2 – 10, 3 – 9, 4 – 8, 5 – SS1 (where SS1 stands for subsequence 1). The resulting present value for this subsequence is +1.92; again, the subsequence is retained in the optimum sequence.

This process is repeated until the first block mined is contained in the subsequence. The final present value calculated is the present value of the deposit. The present value of this hypothetical deposit is +12.60. Fig. 3 shows the final pit outline and mining sequence.

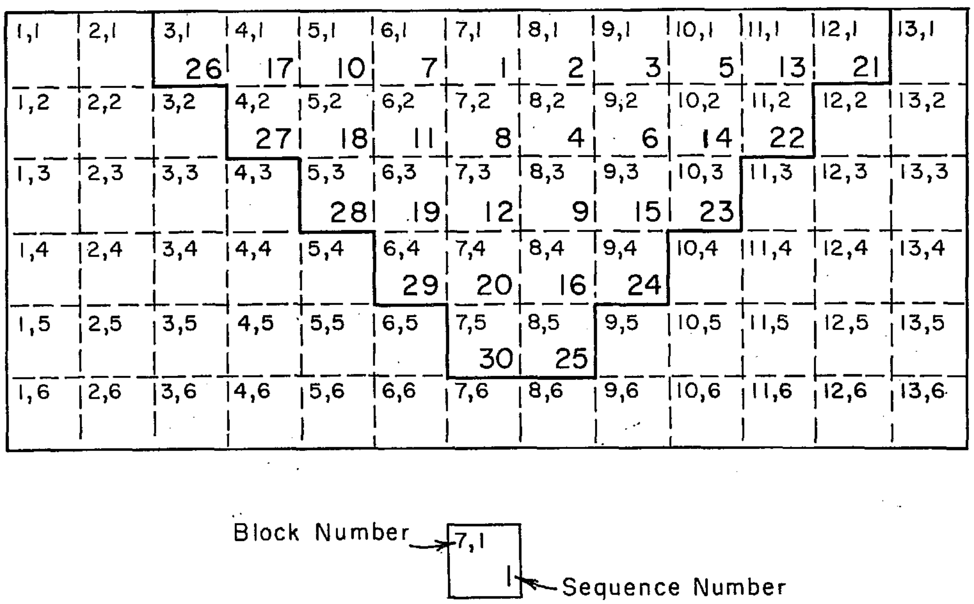

FIGURE 3 – Schematic of the deposit showing final pit outline and optimum sequence for removing blocks.

APPLICABILITY OF SET DYNAMIC PROGRAMMING

The number of sequences to be considered increases as the number of blocks contained in the deposit increases. Even for a small deposit the amount of calculations required is too great to allow the problem to be solved without the aid of a computer. Adding constraints to the mining sequence will reduce the number of sequences possible in any time period. In the example just discussed, only two constraints are on the mining sequence: 1) pit wall slope not to exceed 1:1, and 2) each level would be entered only at one point. By eliminating the second constraint, the problem of sequencing 36 blocks required approximately 30 times more computer time to solve, and 10 times more computer storage – emphasizing the importance of defining the problem.

CONCLUSION

This method for solving the sequence problem in open-pit mining is computationally simple. It is versatile in that the constraints are incorporated in the logic of the subroutine which developed the sequences, and can be changed readily. In addition, the procedure gives the mathematical optimum, not just a good solution. The utility of the procedure depends solely on the availability of good geologic data on the deposit, and the ingenuity of the mining engineer in developing realistic constraints on the allowable mining sequences. As pointed out by Davis and Williams (1973) the economic incentive for developing an optimum mining sequence are great. In their example the pay back period for a medium size copper mine would have been reduced to nearly half if the near-optimum mining sequence had been determined and used.

Some mention of the practical applicability of this dynamic programming technique to a realistic problem is desirable: a 3-dimensional real ore deposit divided into an appropriate number of mining blocks. First, the technique demonstrated is applicable to a 3-dimensional ore deposit. The available computer capacity and skill of the programmer are only limiting factors in the size of the problem to be handled. Second, adding constraints to the acceptable sequences will reduce the size of the problem, and the time required to solve the problem, but may make the job of determining which blocks are not to be mined more difficult. For this reason the constraints on the mining sequence should be selected with great care. In addition, arbitrary selection of constraints to include unnecessary constraints may limit the choice of a sequence to one, which is less profitable than the unbound optimum sequence. The fewer the constraints, the more freedom is allowed in selecting the optimum sequence; consequently, the more profitable the optimum sequence may be. Second, to limit the size of the problem, mining blocks should be selected to be consistent with the amount of geologic data available. Compromises will be necessary to allow for changing pit slopes and smooth pit periphery.

Finally, the procedure outlined is not intended to be used once to give the optimum sequence and that sequence followed for the entire mine life, but, as economic conditions change, as improvements in metallurgy are affected, and as refinements are made in the knowledge of the geology of the deposit, the mining sequence should be redetermined. As a result the optimum mining sequence will be determined for the next 15 years, and this sequence may be adjusted every year or two. This philosophy should prevail whatever procedure is used for determining the mining sequence. The result will be a sequence that is optimum now, from the best information at hand.

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