CIRCULAR 117

Batch-Grinding Model

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NEW MEXICO STATE BUREAU OF MINES AND MINERAL RESOURCES

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New Mexico State Bureau of Mines and Mineral Resources

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BATCH-GRINDING MODEL

by R. J. Roman and G. W. Becker

Socorro

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PREFACE

Beginning in the mid-50's, grinding models have been developed from numerous investigations, the aim of which have been to help the engineer predict the performance of grinding circuits. Another goal has been to offer a model by which grinding circuits may be controlled through the use of an on-line computer. Models presented to date have not been entirely satisfactory as evidenced by the continued work in this area. The model presented in this circular is the result of initial work done at the New Mexico Bureau of Mines and Mineral Resources to develop a realistic grinding model, the logical starting point of which is the development of a batch grinding model. From this point the model can be made more sophisticated by incorporating factors for each of the variables deemed important. Finally, a model of a complete grinding circuit will be developed by combining a model of a continuous grinding mill together with that of a classifier.

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ABSTRACT

A grinding model has been developed which can be used to predict the product size distribution at any retention time in a batch grinding mill, given the feed size distribution and product size distribution at any other retention time. This model differs from other models published in that (1) the breakage function is completely specified through the Schuhmann equation and a mass balance, and (2) the apparent selection function is assumed to be dependent on the size distribution of the material in the mill, as well as the particle size being considered. These two factors have been used to develop a model which can make use of data obtained from grinding a size distribution to predict the results of grinding over a 10-fold change in size modulus.

INTRODUCTION

The development of most grinding models starts with a general equation which relates the mass in any size fraction, by means of a mass balance to the time of grinding, based on the observation that grinding exhibits first-order kinetics. Equation (1) is such an equation.*

$$\frac{dM(i)}{dt} = -S(i)\, M(i) + \sum_{j=1}^{i-1} B(i, j)\, S(j)\, M(j);\ i=1, n-1 \qquad (1)$$

Where

M(i) = the mass in the i^{th} size fraction at any point in time, t.
S(i) = the selection function for i^{th} size fraction – the percentage of the mass removed per unit time.
B(i, j) = the breakage function – that fraction of material broken from the j^{th} size into the i^{th} size fraction.
n = the number of size fractions used in the screen analysis.

The normal convention is to number the size fractions from 1 to n, increasing in the order of decreasing particle size. Therefore, the mass in the largest size fraction will be M(1), and the mass in the finest size fraction will be M(n). M(n) is normally found by difference once all the other M(i)'s have been calculated.

A problem arises in the solution of this general equation in that n-1 equations are needed to describe a system of n size fractions and therefore there are $\sum_{i=1}^{n-1} i$ unknowns to be determined. For example, if 11 size fractions are considered, then 55 unknowns exist (10 S(i)'s and 45 B(i, j)'s). In order to develop enough equations to solve for all the unknowns, some assumptions must be made. These usually differ for each investigator and consequently different models are developed.

The model presented in this paper requires two assumptions:

(1) The Schuhmann equation for size distributions can be used to describe the product from the breakage of a single size fraction;
(2) The selection function is dependent on the size distribution of material in the mill and thus varies with time. This dependence can be described by equation (2).

$$S(i) = S(i)^* \left(1 - \sum_{m=1}^{i-1} M(m)\right);\ i=1, n-1 \qquad (2)$$

where S(i)* – the absolute selection function – is equal to the rate of grinding of the i^{th} size when no particles larger than the i^{th} size fraction exist in the mill.

*See Appendix for a development of equation (1).

Although the equations required for solving all the unknowns follow from the first assumption, the second is needed to make the model fit the experiment data. In the development of the model both assumptions will be incorporated into the original n-1 equations to simplify the mathematics.

Assumption No. 1

The Schuhmann equation can be used to describe the size distribution of broken rock. (equation 3)

$$Y = \left(\frac{X}{k}\right)^{a} \tag{3}$$

where Y = cumulative fraction finer than size X
k = size modulus (a constant for a particular material)
a = distribution modulus (a constant for a particular material)

Consider two consecutive size fractions i and i+1. The mass fraction broken into each of them from size j ($j<i$) can be calculated from equations (4) and (5).

$$B(i,j) = Y_{i-1}-Y_i = \left(\frac{X_{i-1}}{k}\right)^{a} - \left(\frac{X_i}{k}\right)^{a} \tag{4}$$

$$B(i+1,j) = Y_j-Y_{i+1} = \left(\frac{X_i}{k}\right)^{a} - \left(\frac{X_{i+1}}{k}\right)^{a} \tag{5}$$

The ratio of the masses in the two fractions is therefore:

$$\frac{B(j+1,j)}{B(i,j)} = \frac{X_i^{a} - X_{i+1}^{a}}{X_{i-1}^{a} - X_i^{a}} \tag{6}$$

If the sizes chosen are limited to the Tyler series ($\sqrt{2}$ series) then because of the relation

$$X_{i+1} = \frac{1}{\sqrt{2}}\, X_i = \frac{1}{2}\, X_{i-1} \tag{7}$$

equation (6) simplifies to

$$\frac{B(i+1,j)}{B(i,j)} = \left(\frac{1}{\sqrt{2}}\right)^{a} \tag{8}$$

Equation (8) can be used to find any $B(i, j)$ given a value for any one of the other breakage functions for size j.

The law of mass conservation requires that the mass of material broken out of the j^{th} size must equal the mass of material broken into all sizes finer than size j which originates from size j. Stated mathematically this law requires that:

$$S(j)\, M(j) = B(j+1,j)\, S(j)\, M(j) + B(j+2,j)\, S(j)\, M(j) + \ldots \tag{9}$$

which simplifies to

$$1 = B(j+1,j) + B(j+2,j) + \ldots \tag{10}$$

Equation (8) can now be used to rewrite equation (10) in terms of B(j+1, j) to give

$$1 = B(j+1,j)\left(1+\left(\frac{1}{\sqrt{2}}\right)^{a}+\left(\frac{1}{\sqrt{2}}\right)^{2a}+\ldots\right) \tag{11}$$

The sum of this converging geometric series can be written

$$1 = B(j+1, j)\left(\frac{2^{\frac{a}{2}}}{2^{\frac{a}{2}}-1}\right) \tag{12}$$

and therefore:

$$B(j+1, j) = 1-2^{-\frac{a}{2}} \tag{13}$$

Equations (8) and (13) show that the breakage function is dependent only on the distribution modulus. Once the distribution modulus is known, each breakage function can be calculated successively.

Equation (1) can now be rewritten, making use of equations (8) and (13):

$$\frac{dM(i)}{dt} = -S(i)\,M(i) + \sum_{j=1}^{i-1} S(j)\left(\frac{1-2^{-\frac{a}{2}}}{2^{\frac{a}{2}(i-j-1)}}\right)M(j);\ i=1,\ n-1 \tag{14}$$

Equation (14), unlike equation (1), contains only one unknown (S(i)) for each size fraction. Its value can be determined if the feed size analysis, a product size analysis, and the retention time are known.

Assumption No. 2

The model described by equation (14) must be an oversimplification of the grinding process since it does not accurately predict the results of batch grinding. It is recognized that in rod mills, coarse particles shield finer particles from breakage. It seems logical to assume that this is also the case in ball mills, but probably to a lesser degree. If the weight of coarse particles is doubled, it may be assumed that the protection they provide is doubled. This protection will vary from 0 to 100 percent as the amount of coarse material varies from 0 to 100 percent. The apparent selection function, S(i), can then be expressed by equation (15).

$$S(i) = S(i)^*\left(1-\sum_{m=1}^{i-1} M(m)\right);\ i=1,\ n-1 \tag{15}$$

where M(m) is expressed as a decimal percentage of the total feed.[1]

Equation (14) now becomes

$$\frac{dM(i)}{dt} = -S(i)^*\left(1-\sum_{m=1}^{i-1} M(m)\right)+\sum_{j=1}^{i-1} S(j)^*\left(1-\sum_{m=1}^{i-1} M(m)\right)\left(\frac{1-2^{-\frac{a}{2}}}{2^{\frac{a}{2}(i-j-1)}}\right)M(j);\ i=1,\ n-1 \tag{16}$$

This equation can be used to determine the unknowns, S(i)*, by applying Simpson rules. This requires an initial guess for each S(i)*. S(1)* can be determined directly from a feed-screen analysis and a product-screen analysis after one grinding test. The calculated results are then compared with the experimental data and S(2)* is adjusted until a satisfactory fit of size fraction 2 is obtained. Each successive selection function is found in the same manner. Once all the S(i)'s are determined, equation (16) can be solved to give the mass in each size fraction for any length of grinding time.

[1] M(m) can be expressed in units of weights. However, equation 15 then becomes

$$S(i) = S(i)^*\left(\frac{\sum_{m=1}^{n} M(m) - \sum_{m=1}^{i-1} M(m)}{\sum_{m=1}^{n} M(m)}\right)$$

where $\sum_{m=1}^{n} M(m)$ = the total weight of material in the mill.

EXPERIMENTAL VERIFICATION OF THE MODEL

Quartz from the Harding mine near Dixon, New Mexico, was selected for the grinding tests. All of the grinding tests were run in a 12" x 12" Bico ball mill with the ball charge recommended by Bond for work index determinations. The feed was stage crushed in rolls to pass a 6-mesh screen and then split into 1,200-gram samples (equivalent to a volume of 700 cc). The mill speed was 70 rpm.

Duplicate screen analyses were run on 300-gram samples of the feed and product from each grinding test and average results are reported in this paper. The 300-gram samples were first washed of slimes by decantation through a 400-mesh screen. The sands were dried, then screened on a RoTap. The weight of the -400-mesh fraction was obtained by difference. Table 1 lists the screen analyses for the feed and products from 1, 2, 4, 8, 16 and 32 minutes of grinding.

In matching the experimental data to equation (17), n (the number of size fractions) was equal to 14 with the pan (-400-mesh) fraction calculated by difference. The distribution modulus (a) was measured from the size distribution of the product of the 4-minute grind. The value for a used was 0.88. A selection function for the first size fraction (+8-mesh) was determined by measurement from a first-order plot: log percentage of the plus 8-mesh remaining in the product versus time. Initial guesses were then made for the values of S(2)* to S(13)* and the model predictions compared to the experimental data for a 4-minute grind. A computer was used to facilitate the calculations. Because material in the pan fraction that is broken still remains in the pan fraction S(14)* = a S(2)* was then adjusted to give the best agreement with the experimental data for size fraction 2. Next, S(3)* was adjusted until the model prediction agreed with the experimental data for size fraction 3. This process of adjusting the selection function was continued until S(13)* was determined. The final values for the selection functions are shown in Figure 1. Table II lists the predicted screen analyses of products from 1, 2, 4, 8, 16, and 32-minute grinds. Figure 2 shows both the experimental and predicted size analyses of the different grinds.

The selection functions appear to obey the relation

$$\frac{S(i)^*}{S(i+1)^*} = \left(\frac{\overline{X}(i)}{\overline{X}(i+1)} \right)^{\alpha} \qquad (17)$$

for the finer sizes, where X(i) = average particle size in the i^{th} fraction. It also appears that the selection function goes through a maximum. This suggests the existence of an optimum particle size which may be a function of the grinding equipment, material being ground, or both.

CONCLUSIONS

The model for batch grinding proposed appears to accurately predict the product size analysis for batch grinding over a 10-fold change in size modulus. The data needed to determine the constants for the model are data that would normally be available or could be easily obtained from plant data. The application of this model to continuous grinding appears feasible and work is underway to demonstrate this. A major obstacle is translating results from a lab-size grinding mill to full size mills. Because of this, an effort will be made to keep the model in a form which can use plant data.

TABLE 1. SIZE ANALYSES OF FEED AND PRODUCTS OF GRINDING TESTS

		Cumulative % Passing					
Mesh	Feed	1 min.	2 min.	4 min.	8 min.	16 min.	32 min.
8	87.71	93.46	96.31	98.63	99.69		
10	74.10	83.04	89.79	96.56	99.29		
14	61.15	72.12	83.74	94.71	99.02	99.69	
20	49.06	60.69	75.68	91.90	98.68	99.54	
28	39.06	49.63	65.91	86.87	98.16	99.36	
35	30.61	39.57	54.55	77.65	96.71	99.15	98.95
48	23.86	30.95	43.80	65.42	91.75	98.89	98.69
65	18.10	23.62	33.82	52.45	81.06	97.92	98.30
100	13.32	17.59	25.15	40.01	65.60	93.17	98.09
150	9.65	12.95	18.39	29.61	50.56	80.92	97.07
200	7.00	9.77	13.58	22.17	39.15	67.27	93.46
270	5.28	7.69	10.49	17.20	31.02	55.79	85.96
400	3.43	5.31	6.93	11.57	21.32	40.78	70.03

TABLE 2. MODEL-PREDICTED SIZE ANALYSES OF PRODUCTS FROM GRINDING TESTS

	Cumulative % Passing					
Mesh	1 min.	2 min.	4 min.	8 min.	16 min.	32 min.
8	93.79	96.23	98.63			
10	83.63	89.77	96.12			
14	74.52	83.79	93.79	99.14		
20	64.46	76.36	90.48	98.65		
28	54.26	67.51	85.54	97.74		
35	43.82	56.56	76.86	94.95		
48	34.62	45.79	65.94	89.40	99.26	
65	26.50	35.59	53.60	79.76	97.01	
100	19.71	26.81	41.70	67.07	91.36	99.48
150	14.43	19.83	31.56	53.80	81.95	97.72
200	10.53	14.56	23.48	41.49	69.00	92.54
270	7.87	10.84	17.51	31.49	55.64	83.73
400	5.32	7.50	12.43	23.03	42.74	71.04

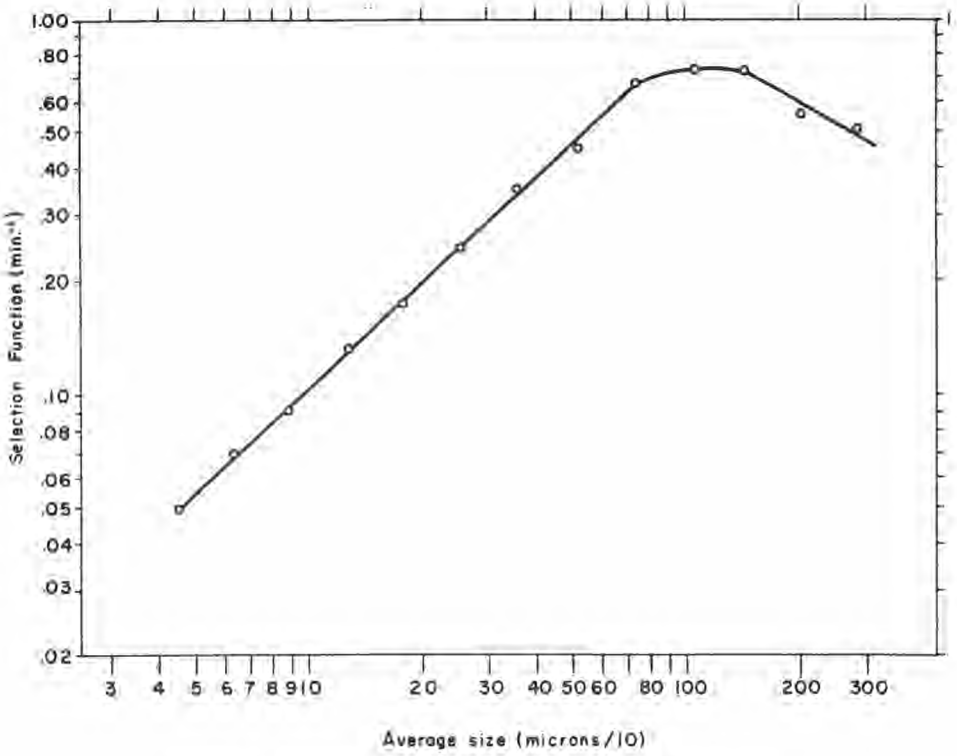

Figure 1. Selection function versus average particle size.

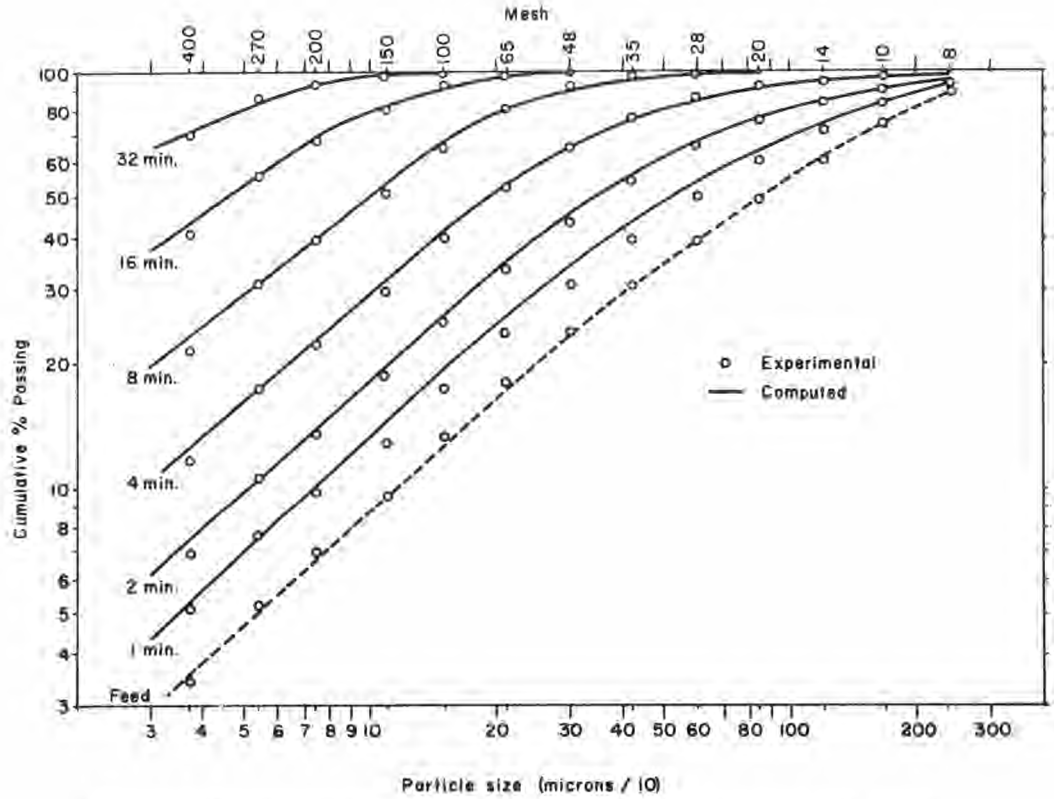

Figure 2. Experimental and predicted product size analyses for batch grinding.

APPENDIX

It seems reasonable to assume that grinding is a first-order process, that is, the rate at which any size fraction grinds is directly proportional to the mass of that size fraction in the mill, other variables being held constant. This may be expressed mathematically as

$$\frac{dM(i)}{dt} = -S(i)\ M(i)$$

where $\frac{dM(i)}{dt}$ = the rate of change of size i with time, i.e. - rate of grinding

M(i) = instantaneous mass of size i

S(i) = rate constant for size i.

When more than one size fraction exists in the mill, the above equation must be modified to include terms which represent the increase in mass in size fraction i, due to particles larger than i being broken into i. Assume, now, an arbitrary number of size fractions, n. The charge of mass of largest size fraction can be described by the above equation since there are no particles larger which will be broken into this fraction. The second size fraction, however, will receive material from the first size fraction as grinding progresses. This can be expressed mathematically as

$$\frac{dM(2)}{dt} = -S(2)\ M(2) + B(2, 1)\ S(1)\ M(1)$$

where B(2, 1) = that fraction of mass broken from the first size which reports to the second size.

S(2) and S(1) = rate constants for the second and first sizes respectively.

The third size fraction can be described by the equation

$$\frac{dM(3)}{dt} = -S(3)\ M(3) + B(3,1)\ S(1)\ M(1) + B(3, 2)\ S(2)\ M(2)$$

The general equation for the i^{th} size fraction therefore is

$$\frac{dM(i)}{dt} = -S(i)\ M(i) + \sum_{j=1}^{i-1} B(i, j)\ S(j)\ M(j)$$

where the 1st term on the right hand side of the equation represents the loss of mass due to material being broken from size i and the second term represents the gain in mass due to the larger particles being broken into size i.